OCR
Oxford Cambridge and RSA

Wednesday 28 June 2017 – Morning

A2 GCE MATHEMATICS

4734/01 Probability & Statistics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4734/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Answer **all** the questions.

1 The random variable X has the distribution $N(\mu, 3^2)$. A random sample of 9 observations of X produced the following values.

6 2 3 6 8 11 12 5 10

(i) Find a 90% confidence interval for μ. **[4]**

(ii) Explain what is meant by a 90% confidence interval in this context. **[1]**

2 In a random sample of 40 female students, 32 passed a particular examination. In a random sample of 34 male students, 25 passed the same examination. Test at the 5% significance level whether the proportion of females passing the examination differs from the proportion of males passing the examination. **[6]**

3 The hair colour and eye colour of 100 randomly selected people were noted. The results are shown in the table.

	Dark hair	Not dark hair
Brown eyes	36	22
Not brown eyes	16	26

Use a χ^2 test at the 5% significance level to test whether there is an association between hair colour and eye colour. **[7]**

4 X, Y and Z are random variables. X and Y have independent Poisson distributions with means 2 and 3 respectively, and $Z = 4X + 5Y$.

(i) Find $E(Z)$ and $Var(Z)$. **[3]**

(ii) Explain how your answers to part **(i)** show that Z does not have a Poisson distribution. **[1]**

(iii) Find $P(Z = 15)$. **[4]**

5 A greengrocer sells apples whose masses are normally distributed. The greengrocer claims that the mean mass of the apples is at least 180 grams. A shopper buys a random selection of 8 apples from the greengrocer. The masses of these apples, in grams, are as follows.

160 200 164 170 186 192 162 178

These 8 masses were used to calculate an unbiased estimate of the population variance, and the value was found to be 223.714, correct to 3 decimal places. Test at the 1% significance level whether there is any evidence to doubt the greengrocer's claim. **[7]**

6 Each time a motorist refuels her car, she calculates the average rate of fuel consumption C, in km/l, since the last time she refuelled the car. She investigates whether C has a normal distribution. She chose 60 of her calculations at random, and the results are summarised in the table.

C (km/l)	< 16.5	16.5 – 17.0	17.0 – 17.5	17.5 – 18.0	18.0 – 18.5	18.5 – 19.0	19.0 – 19.5	> 19.5
Observed frequency	0	8	17	12	11	8	4	0

(i) Show that an estimate of the mean fuel consumption is 17.8 km/l. **[2]**

The standard deviation of the data in the table was calculated and was found to be 0.7286 km/l, correct to 4 decimal places. Using these values for the mean and standard deviation, the expected frequencies for the eight classes were found, and these are shown in the table.

C (km/l)	< 16.5	16.5 – 17.0	17.0 – 17.5	17.5 – 18.0	18.0 – 18.5	18.5 – 19.0	19.0 – 19.5	> 19.5
Expected frequency	2.23	5.93	12.25	16.07	13.42	7.11	2.40	0.59

(ii) Show how the expected value of 12.25 for the 17.0 – 17.5 class was calculated. **[5]**

(iii) Carry out a χ^2 goodness of fit test at the 5% level of significance. **[7]**

7 A continuous random variable X has probability density function

$$\mathrm{f}(x) = \begin{cases} kx^2 & 0 \leqslant x \leqslant 2, \\ kx^2 - 1.5x + 3 & 2 < x \leqslant 4, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{16}$. **[2]**

(ii) Find $\mathrm{P}(1.5 \leqslant X \leqslant 3)$. **[3]**

(iii) Show that the upper quartile of X is 2.41, correct to 2 decimal places. **[4]**

8 Ten randomly selected joints of meat, A, B, …, J, are each cut in half. One half of each joint of meat is frozen and packed using process 1 and the other half is frozen and packed using a new process 2. All the halves are then placed in the same freezer. For each pack, the number of days that the meat takes to spoil is found. The results are shown in the table.

Joint	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Process 1	78	108	80	171	184	153	50	145	91	156
Process 2	111	106	77	196	230	148	58	155	90	170

Carry out a paired sample t-test, at the 5% significance level, to investigate whether meat frozen and wrapped using process 2 takes longer to spoil than meat frozen and wrapped using process 1. State any necessary assumption. **[10]**

9 The object distance U and the image distance V for a convex lens are variables. They are related to the focal length f of the lens, which is a constant, by the formula

$$\frac{1}{U}+\frac{1}{V}=\frac{1}{f}.$$

U is a random variable with a continuous uniform distribution over the interval from $3f$ to $4f$. Find the probability density function of V. **[6]**

END OF QUESTION PAPER

Question			Answer	Marks	Guidance	
1	(i)		$\bar{x} = 7$ $"7" \pm z\frac{3}{\sqrt{9}}$ $7 \pm 1.645\frac{3}{\sqrt{9}}$ [5.355, 8.645]	B1 M1 A1 A1 **[4]**	NOT t Accept 5.35/6 and 8.64/5	Might use 1.6449
	(ii)		90% of such intervals will contain μ.	B1 **[1]**	Must refer to more than 1 CI.	
2			$\hat{p} = \frac{57}{74} = 0.770$ $H_0: p_f = p_m$ $H_1: p_f \neq p_m$ $\frac{\frac{32}{40} - \frac{25}{34}}{\sqrt{\left(\frac{57}{74} \times \frac{17}{74} \times \left(\frac{1}{40} + \frac{1}{34}\right)\right)}}$ 0.659 -1.96 < "0.659" < 1.96, do not reject H_0 There is insufficient evidence of a difference between (popn) proportions of females and males passing the exam.	B1 B1 M1 A1 M1 A1 **[6]**	Var=$\frac{57}{5920}$ Allow 1 correct inequality. cwo. Contextualised, not over-assertive.	Not pooled. B0,B1 (hyps), 0.656 B1, M1A0 3/6

Question			Answer	Marks	Guidance	
3			H_0: there is no assoc between hair/eyes colours. H_1: there is assoc	B1		
			Es 30.16, 27.84, 21.84, 20.16	B1		
			$(\|36 - 30.16\| - 0.5)^2/30.16$ + ...	M1	allow this mark if no Yates' correction.(5.61)	0.945+1.306+1.024+1.414
			4.69	A1		
			CV = 3.841	B1		
			4.69 > 3.841, reject H_0,	M1	ft TS and CV	
			There is evidence of an assoc. between hair/eye colours.	A1 **[7]**	cwo. Contextualised.	
4	(i)		23; $4^2\times2 + 5^2\times3$	B1;M1	32 or 75 oe seen M1	
			= 107	A1 **[3]**		
	(ii)		E(Z)≠Var(Z)	B1 **[1]**	OR only X + Y ~ Po.	
	(iii)		(0,3), identified	B1	allow '0' missing for this mark.	
			$\frac{e^{-2}\times2^0}{0!}$ or $\frac{e^{-3}\times3^3}{3!}$	M1		
			Both, multiplied	M1	fully correct method.	0.1353352x0.2240418
			0.0303	A1 **[4]**		
5			$\bar{x} = 176.5$	B1	Can be implied by (-)3.5 later	
			H_0: $\mu = 180$ H_1: $\mu < 180$	B1	Allow $\mu \geq 180$	
			$t = \frac{176.5-180}{\frac{14.957}{\sqrt{8}}}$	M1		
			= -0.662	A1		
			CV = -2.998	B1	or p=0.265	

Question			Answer	Marks	Guidance	
			-0.662 > -2.998, do not reject H_0, There is insufficient evidence to doubt the greengrocer's claim.	M1 A1 **[7]**	TS > CV, their values cwo. Contextualised, not over-assertive.	0.265>0.01 M1
6	(i)		$\frac{16.75\times 8+...+19.25\times 4}{60}$ 17.8 AG	M1 A1 **[2]**	1068/60	
	(ii)		$\frac{17.5-17.8}{0.7286}$ = -0.412, $\frac{17.0-17.8}{0.7286}$ = -1.098 Φ(-0.412) – Φ(-1.098) 60(0.3402 – 0.1360) 12.25 AG	M1 A1 M1 M1 A1 **[5]**	standardise at least once either -0.412 or -1.098 attempt to find Φ(...) of either value subtract and multiply by 60.	 0.2042 or 0.8640-0.6598
	(iii)		<17.0 17.0-17.5 17.5-18 18.18.5 >18.5 8 17 12 11 12 8.16 12.25 16.07 13.42 10.1 H_0: data can be modelled by N H_1: data cannot be modelled by N $\frac{(8-8.16)^2}{8.16}$ ..+.... = 3.67 3.67 < 5.991, do not reject H_0, Insufficient evidence that the data cannot be modelled by a normal distribution.	B1 B1 B1 M1 A1 M1 A1 **[7]**	Can be implied both rows correct ft df=classes-3 cwo	Correct combination of classes. 8 groups lead to 8.03
7	(i)		$\int_0^2 kx^2 dx = 0.5$ oe	M1	Distribution is symmetrical.	

Question			Answer	Marks	Guidance	
			$= \frac{3}{16}$ AG	A1 **[2]**		
	(ii)		$\int_{1.5}^{2} \frac{3x^2}{16} dx + \int_{2}^{3} (\frac{3x^2}{16} - \frac{3x}{2} + 3) dx$ $\left[\frac{x^3}{16}\right]_{1.5}^{2} + \left[\frac{x^3}{16} - \frac{3x^2}{4} + 3x\right]_{2}^{3}$ $= 0.727 = \frac{93}{128}$	M1 B1 A1 **[3]**	 ignore limits for this mark	
	(iii)		$\int_{2}^{U} \left(\frac{3x^2}{16} - \frac{3x}{2} + 3\right) dx = 0.25$ oe $U^3 - 12U^2 + 48U - 60 = 0$ oe 2.405 → -0.0577 2.415 → 0.018 change of sign, $U = 2.41$ AG	M1 A1 M1 A1 **[4]**	 if GC need answer to 3 or more dp. 2.412598948	 or Newton-Raphson or iteration $\int_{2.41}^{4} f(x)\,dx$=0.2512 SCB2
8			d: 33 -2 -3 25 46 -5 8 10 -1 14 mean = 12.5, sd = 17.213 H_0: $\mu_D = 0$, H_1: $\mu_D > 0$ $\frac{12.5}{\frac{17.213}{\sqrt{10}}}$ = 2.296 CV = 1.833 2.296 > 1.833, reject H_0, There is sufficient evidence that process 2 is an improvement. Differences are normally distributed.	B1 B1B1 B1 M1 A1 B1 M1 A1 B1	. can be implied by correct mean, sd. Allow 134.1-121.6 ; s^2=296.3 or 5333/18 Allow $\mu_1=\mu_2$ etc. Allow in words unless clearly referring to sample. Allow 2.30 TS > CV ft. cwo. Contextualised, not over-assertive.	Not paired. B0B1B0B1M0A0B1(1.734 or 1.833)M1A0B0 4/10max

Question			Answer	Marks	Guidance	
				[10]		
9			Pdf of U=$\frac{1}{f}$ $V = \frac{Uf}{U-f}$ or $U = \frac{Vf}{V-f}$ $\int_{3f}^{u} \frac{1}{f}\,du = \frac{u}{f} - 3$ cdf of V = P($\frac{fU}{U-f} \le v$) 1-($\frac{vf}{v-f} - 3$) oe differentiate $\frac{f}{(v-f)^2}$ $\frac{4f}{3} \le v \le \frac{3f}{2}$	B1 B1 B1 M1 A1 B1 **[6]**	$\int_{3f}^{4f} \frac{1}{f}du = 1$ and attempt at int. by subn. M1 $\frac{du}{dv} = \frac{-f^2}{(v-f)^2}$ B1 $\int_{\frac{4f}{3}}^{\frac{3f}{2}} \frac{f}{(v-f)^2}dv = 1$ so pdf of v is $\frac{f}{(v-f)^2}$ A1 range B1	